# Spacetime structure of 5D hypercylindrical vacuum solutions with tension

Inyong Cho\*

BK21 Physics Research Division and Department of Physics, Sungkyunkwan University, Suwon 440-746, Korea

Gungwon Kang<sup>†</sup>

Korea Institute of Science and Technology Information, 52-11 Eoeun-dong, Yuseong-gu, Daejeon 305-806, Korea

Sang Pvo Kim<sup>‡</sup>

Department of Physics, Kunsan National University, Kunsan 573-701, Korea

Chul H. Lee§

Department of Physics and BK21 Division of Advanced Research and Education in Physics, Hanyang University, Seoul 133-791, Korea

We investigate geometrical properties of 5D cylindrical vacuum solutions with a transverse spherical symmetry. The metric is uniform along the fifth direction and characterized by tension and mass densities. The solutions are classified by the tension-to-mass ratio. One particular example is the well-known Schwarzschild black string which has a curvature singularity enclosed by a horizon. We focus mainly on geometry of other solutions which possess a naked singularity. The light signal emitted by an object approaching the singularity reaches a distant observer with finite time, but is infinitely red-shifted.

PACS numbers: 04.50.+h,04.70.-s

Keywords: Black string, Singularity, Causal structure

#### I. INTRODUCTION

Black string solutions are higher dimensional black hole spacetimes possessing "hypercylindrical" horizons with/without compactification, instead of "spherical" ones. Recently, it has been of much interest studying properties of those spacetime backgrounds. Being different from that the stationary black hole with a spherical horizon topology is stable, the Schwarzschild black string was found to be unstable under small perturbations; it is the so-called Gregory-Laflamme (GL) instability [1]. This instability has been studied in many ways afterwards. In particular, whether a perturbed black string is fragmented into an array of small black holes, or ends up with a stable non-uniform black string has been a hot issue [2, 3]. The robustness of the GL instability has also been studied in supergravity theories [4] as well as in general relativity with a negative cosmological constant [5]. However, it is still not understood well what really causes the GL instability.

The Schwarzschild black string is a particular case of the 5D hypercylindrical vacuum solution. It is characterized by a single parameter (usually called M) while the general solution has two parameters. The two-

\*Electronic address: iycho@skku.edu
†Electronic address: gwkang@kisti.re.kr
‡Electronic address: sangkim@kunsan.ac.kr
§Electronic address: chulhoon@hanyang.ac.kr

parameter solution was first found by Kramer [6] and was manipulated in various ways by others in the literature [7, 8, 9, 10, 11]. Although geometrical propeties of this spacetime were studied in many works, most of the studies were in the context of the Kaluza-Klein thoery. Consequently, understanding of the geometry was based on the four-dimensional gravity with a scalar field. This caused many misleading interpretations for the full five-dimensional geometry of the solutions.

Very recently, the physical meaning of the two parameters was correctly interpreted for the first time in Ref. [11] by one of us. The author considered the weak-field solutions of the Einstein field equations outside some matter distribution. He identified the two parameters with "mass" and "tension" densities by matching with the vacuum solutions at asymptotic region. When the tension-to-mass ratio is exactly one half, in particular, it corresponds to the Schwarzschild black string.

In this paper we investigate geometrical properties of such "hypercylindrical" spacetime having arbitrary tension in detail. The solutions are classified mainly by the tension-to-mass ratio a. The physical range of the ratio is  $0 \le a \le 2$ . Specific values a = 1/2 and a = 2 correspond to the well-known Schwarzschild black string and Kaluza-Klein bubble [12]. We are particularly interested in the other values of a in the range. The geometry possesses a naked singularity. The light signal emitted by an observer approaching the singularity escapes within finite time, but is infinitely redshifted. There is no wormhole structure in the full five dimensional geometry.

In Section II, we study the geometrical properties of

the hypercylindrical solution. In Section III, we discuss the causal structure of the spacetime, and we conclude in Section IV.

### II. GEOMETRICAL PROPERTIES

The most general form of the static metric for the transverse spherically symmetric static spacetime with a translational symmetry along the fifth spatial direction in five dimensions may be written as

$$ds^2 = -Fdt^2 + G\left[d\rho^2 + \rho^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] + Hdz^2.(1)$$

Here F, G and H are functions of the "isotropic" radial coordinate  $\rho$  only. Note that the fifth direction is not assumed to be flat in general, i.e.,  $H \neq 1$ . If we include a constant momentum flow along the z direction, the  $g_{tz}$  component is not zero in general. Such a stationary solution was considered in Refs. [7, 13]. Time-dependent solutions in a separable form were found in Ref. [14]. A class of solutions allowing the z-dependence was also considered in Ref. [15]. By taking double Wick rotations, i.e.,  $t \to iz$  and  $z \to it$  in the metric (1), one can easily see that, given a solution with F and H, the metric with F and H being exchanged is a solution as well.

The hypercylindrical type of system has been studied by a number of people, and its vacuum solution has been obtained in various forms. The solutions are basically two-parameter solutions. The interpretation of these two integration constants has not been given properly for a long time. In Ref. [7] these constants were related to the gravitational mass and scalar charge. Davidson and Owen [9] defined two kinds of mass parameters in the context of Kaluza-Klein dimensional reduction. Namely, they defined the gravitational mass parameter by considering the asymptotic behavior of the four-dimensional effective metric, and speculated that the other mass parameter is somehow related to the Kaluza-Klein electric charge.

It is Ref. [11] in which the physical meaning of these integration constants was correctly given. Using the metric ansatz (1), the solutions are given by

$$F(\rho) = \left| \frac{1 - \frac{K}{\rho}}{1 + \frac{K}{\rho}} \right|^{\frac{2(2-a)}{\sqrt{3(a^2 - a + 1)}}}, \tag{2}$$

$$G(\rho) = \left(1 + \frac{K}{\rho}\right)^4 \left| \frac{1 + \frac{K}{\rho}}{1 - \frac{K}{\rho}} \right|^{\frac{2(a+1)}{\sqrt{3(a^2 - a + 1)}} - 2}, \quad (3)$$

$$H(\rho) = \left| \frac{1 - \frac{K}{\rho}}{1 + \frac{K}{\rho}} \right|^{\frac{2(2a-1)}{\sqrt{3(a^2 - a + 1)}}}.$$
 (4)

In Ref. [11], the author considered weak-field nonvacuum solutions of five dimensional Einstein equations for a system of matter distribution characterized by the mass and tension densities having the same spherical and translational symmetries mentioned above. By comparing them

with the asymptotic behaviors of the metric components above at spatial infinity  $(\rho \gg K)$ , he found the relationship between these two integration constants and the physical parameters of the linear-mass density  $\lambda$  and the linear-tension density  $\tau$ ,

$$a = \frac{\tau}{\lambda},\tag{5}$$

$$K = \sqrt{\frac{1 - a + a^2}{3}} G_5 \lambda. \tag{6}$$

Actually, this identification is a sort of analogy in the sense that the internal-vacuum region is replaced by a compact matter having the same symmetries. Therefore, the mass and tension in this analogy are contributions from the matter stress-energy inside, but are not pure gravitational contributions. The rigorous definitions of mass and tension densities for gravitational fields themselves can be found in Refs. [16, 17, 18] where the ADM-tension density is associated with the asymptotic spatial-translation symmetry along the z direction in much the same way as that the ADM-mass density is associated with the asymptotic time-translation symmetry. Such definitions give the same relationship above.

The geometrical properties of the spacetime under consideration are very different depending on the value of the tension-to-mass ratio a. (K can be absorbed to the radial coordinate  $\rho$ .) The particular case of a=1/2 stands for the well-known Schwarzschild black string,

$$ds^{2} = -\left(\frac{1 - K/\rho}{1 + K/\rho}\right)^{2} dt^{2}$$

$$+ \left(1 + \frac{K}{\rho}\right)^{4} (d\rho^{2} + \rho^{2} d\Omega_{2}^{2}) + dz^{2}, \qquad (7)$$

$$= -\left(1 - \frac{2M}{r}\right) dt^{2}$$

$$+ \left(1 - \frac{2M}{r}\right)^{-1} dr^{2} + r^{2} d\Omega_{2}^{2} + dz^{2}, \qquad (8)$$

where  $M=2K=G_5\lambda$  and  $r=\rho(1+K/\rho)^2$  is the usual "isentropic" circumference radius. The case of a=2 stands for the so-called Kaluza-Klein bubble [12].

A priori there is no restriction on the parameter values. However, if we assume the positivity of the mass density, we have  $\lambda \geq 0$ . In Ref. [19] it was proved that the pure gravitational contribution to the tension is positive definite. Therefore, the physical range for the values of the tension would be  $\tau \geq 0$  (i.e.,  $a \geq 0$ ). From the weak-field approximation studied in Ref. [11], in a relation to the matter contribution to the tension, one can show that there exists an upper bound in the tension provided that the matter satisfies the strong-energy condition. This upper bound gives  $\tau \leq 2\lambda$  (i.e.,  $a \leq 2$ ) in five dimensions. We apply the same upper bound for the gravitational contribution to the tension although there is no definite proof for it in the literature as long as we know. Therefore, one may assume that the physical range

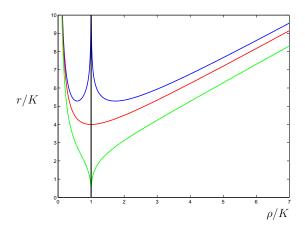


FIG. 1: Plot of r vs.  $\rho$ . From the top down, the curves show the typical shape corresponding to the case of 1/2 < a < 2, a = 1/2, 2, and 0 < a < 1/2.

of the tension parameter is

$$0 \le \tau \le 2\lambda$$
 (i.e.,  $0 \le a \le 2$ ). (9)

Based on some desirable cosmological behavior, Davidson and Owen [9] speculated that the physical choice is a < 1/2. On the other hand, Ponce de Leon [20] claimed that -1 < a < 1/2 by imposing the physical energy conditions on the four-dimensional effective matter induced in the Kaluza-Klein dimensional reduction.

Regarding the geometrical properties, first note that the Kretschmann scalar  $R_{ABCD}R^{ABCD}$  diverges at  $\rho = K$  except for a = 1/2 and a = 2. Therefore, a curvature singularity locates there.

Now let us consider the isentropic radius r defined by

$$r = \rho \sqrt{G(\rho)} = \frac{(\rho + K)^2}{\rho} \left| \frac{\rho + K}{\rho - K} \right|^{\frac{(a+1)}{\sqrt{3(a^2 - a + 1)}} - 1}.$$
 (10)

The shape of  $r(\rho)$  is classified into three types depending on the scale of a as shown in Fig. 1. As we can see from the figure, there exist two copies of the spacetime separated by  $\rho = K$ . Both copies share the same geometry since the metric is invariant under the transformation  $\rho \to K/\rho^2$ . Therefore, we shall focus on the region of  $\rho \geq K$  from now.

For the Schwarzschild black string (a=1/2), the  $\rho=K$  surface corresponds to the event horizon at r=2M (= 4K) from Eq. (8). The  $\rho$ -coordinate cannot describe the interior region of the horizon while the r-coordinate can do. The Kaluza-Klein bubble (a=2) solution is related to the Schwarzschild black string simply by double Wick rotations.

For 0 < a < 1/2, the space for  $r = [0, \infty)$  is well described by  $\rho = [K, \infty)$ , and those two radial coordinates have a one-to-one correspondence.

For 1/2 < a < 2, the relation between r and  $\rho$  is very peculiar. For a given value of r,  $\rho$  is double valued. There

exists an extremum at

$$\rho_{+} = \frac{a+1+\sqrt{(2a-1)(2-a)}}{\sqrt{3(a^2-a+1)}}K.$$
 (11)

The region below  $r(\rho_+)$  is not covered by the  $\rho$ -coordinate. The  $S^2$  surface area at z= constant hypersurface is  $A(\rho)=4\pi r^2$ . For 1/2 < a < 2, this area is infinite at  $\rho=K$ , reaches minimum at  $\rho=\rho_+$ , and then increases again. Therefore, it looks like a wormhole geometry at  $\rho_+$  and attracted people's attention in earlier works.

The proper length along the fifth direction is given by

$$L(\rho) = \int_{z_0}^{z_0+1} \sqrt{H} dz = \left| \frac{1 - \frac{K}{\rho}}{1 + \frac{K}{\rho}} \right|^{\frac{2a-1}{\sqrt{3(a^2 - a + 1)}}}.$$
 (12)

As  $\rho$  decreases, it monotonically shrinks down to zero at  $\rho = K$  for a > 1/2 whereas it expands to infinity for a < 1/2.

The total surface area of a cylindrical sector having a coordinate distance  $\Delta z = 1$  is then given by

$$A_{\text{total}}(\rho) = A(\rho) \times L(\rho) = 4\pi r^2 \sqrt{H}$$
$$= 4\pi \frac{(\rho + K)^4}{\rho^2} \left| \frac{1 - \frac{K}{\rho}}{1 + \frac{K}{\rho}} \right|^{2 - \frac{3}{\sqrt{3(a^2 - a + 1)}}}. (13)$$

Since the exponent above is always positive except for a=1/2, the total area turns out to be zero at  $\rho=K$  for  $a\neq 1/2$ , and increases monotonically as  $\rho$  increases. For a=1/2, the exponent becomes zero and the surface area is finite. Therefore, although the submanifold at z= constant looks like a wormhole geometry as explained above, the full geometry including the z direction is not that of a wormhole spacetime.

## III. CAUSAL STRUCTURE

In this section we discuss the causal structure of the given geometry. The interesting things would occur at  $\rho = K$  where the singularity is located, and at  $\rho_+$  where the  $S^2$  surface area becomes minimum.

Let us consider radial motions at a z= constant submanifold. The metric becomes

$$ds^{2} = -Fdt^{2} + Gd\rho^{2} = -F(dt + d\rho^{*})(dt - d\rho^{*}),$$
 (14)

where the tortoise coordinate  $\rho^*$  is defined as

$$d\rho^* = \sqrt{\frac{G}{F}} d\rho. \tag{15}$$

The ingoing- and outgoing-null coordinates are defined respectively as

$$v = t + \rho^*$$
 and  $u = t - \rho^*$ . (16)

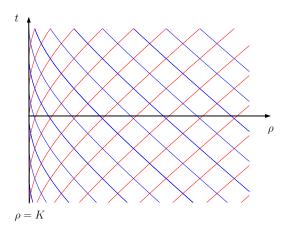


FIG. 2: Plot of ingoing- and outgoing-null geodesics for -1 < a < 2 ( $a \neq 1/2$ ). The null rays touch  $\rho = K$  at a finite time. The geodesics were numerically integrated.

In the vicinity of  $\rho = K$  we have

$$\sqrt{\frac{G}{F}} \sim \left| \rho - K \right|^q, \tag{17}$$

where

$$q = 1 - \frac{\sqrt{3}}{\sqrt{a^2 - a + 1}} \Rightarrow \begin{cases} q = 1 & (a = 1/2), \\ -1 < q < 1 & (a \neq 1/2). \end{cases}$$
 (18)

Therefore, the tortoise coordinate becomes

$$\rho^* \sim \ln |\rho - K| \quad (a = 1/2), \qquad (19)$$

$$\sim |\rho - K|^{q+1} \quad (a \neq 1/2). \qquad (20)$$

For the Schwarzschild black string (a = 1/2),  $\rho^*$  goes to

negative infinity as  $\rho \to K$ . Consequently, from Eq. (16), the ingoing-null geodesic (v = constant) touches the  $\rho = K$  surface at  $t = \infty$ , indicating an event horizon there.

For  $a \neq 1/2$ , however,  $\rho^*$  is finite as  $\rho \to K$ . It implies that the value of t along the ingoing-null geodesic becomes finite on reaching  $\rho = K$ . This indicates that the coordinate t is not singular there as can be seen in Fig. 2. (Note that the proper time measured by a fixed observer outside is proportional to t.)

In order to see whether or not the null rays can escape from the  $\rho = K$  surface, let us now consider the behavior of an outgoing-null geodesic in  $(v, \rho)$  coordinates,

$$ds^{2} = -Fdt^{2} + Gd\rho^{2} = -Fdv\left(dv - 2\sqrt{\frac{G}{F}}d\rho\right). \tag{21}$$

Integrating the outgoing-null geodesic  $du = dv - 2\sqrt{G/F}d\rho = 0$  from  $\rho = K + \epsilon$ , the value of v (a well-defined time coordinate) elapsed for reaching  $\rho = \rho_0$  becomes

$$\Delta v = \int dv = 2 \int_{K+\epsilon}^{\rho_0} \sqrt{\frac{G}{F}} d\rho \tag{22}$$

$$\sim \left\{ \begin{array}{l} \ln \left| \frac{\rho_0 - K}{\epsilon} \right| & (a = 1/2), \\ \left| \rho_0 - K \right|^{q+1} - \left| \epsilon \right|^{q+1} & (a \neq 1/2). \end{array} \right.$$
 (23)

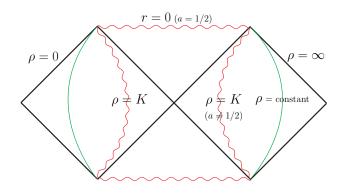


FIG. 3: Penrose diagram at a z = constant slice.

Note that  $\Delta v \to \infty$  as  $\epsilon \to 0$  for the case of a=1/2. It implies the existence of an event horizon at  $\rho=K$ , which is well-known for the case of the Schwarzschild black string. On the other hand,  $\Delta v$  is finite as  $\epsilon \to 0$  for the case of  $a \neq 1/2$ . It implies that the light signal can actually escape from the  $\rho=K$  surface within fine time as it was discussed earlier. Therefore,  $\rho=K$  is not an event horizon, but a naked singularity in this family of spacetime solutions. The global causal structure at a z= constant slice is illustrated in its Penrose diagram in Fig. 3.

Although the light signal takes a finite time to escape from the singularity, the lapse function in the metric (21) vanishes there, which indicates it is an infinite-redshift surface. Therefore, the light signal emitted from the singularity gets infinitely redshifted [21].

The next question is whether or not the singularity at  $\rho = K$  is naked. Usually one easy way to search an event horizon in a spherically symmetric system is to look up where the  $g_{rr}$  component diverges in isentropic coordinates. According to the coordinate transformation (10), the spherically symmetric metric of the 3D sector becomes

$$ds_3^2 = G(d\rho^2 + \rho^2 d\Omega_2^2) = \frac{dr^2}{\left(1 + \frac{\rho}{2G} \frac{dG}{d\rho}\right)^2} + r^2 d\Omega_2^2.$$
 (24)

After some algebra, we can show that the place where the  $g_{rr}$  component diverges corresponds to  $\rho=\rho_+$  defined in Eq. (11). However, it is easy to see from Eq. (14) that this  $\rho_+$  surface is not a null surface. Therefore,  $\rho=\rho_+$  is not an event horizon [23]. It is not very difficult to see that there exists no other candidate for the event horizon in this geometry. Therefore, the  $\rho=K$  surface is a naked singularity.

## IV. CONCLUSIONS

In this paper, we studied a five dimensional cylindrical system in vacuum. We assumed the translational symmetry along the fifth direction and the spherical symmetry along transverse 4D sectors. The solutions to the

Einstein equations of such a system have two parameters which correspond to mass and tension densities. The solutions are classified by the tension-to-mass ratio  $a = \tau/\lambda$ . We concluded that the physically meaningful range is  $0 \le a \le 2$  while particular values correspond to the Schwarzschild black string (a = 1/2) and the Kaluza-Klein bubble (a = 2).

Adopting a metric ansatz in isotropic coordinates for the 3D sector, there exist two identical copies of the spacetime. We investigated interesting geometrical features compared with those of the Schwarzschild black string. Except for a = 1/2 and 2, there exists a curvature singularity at  $\rho = K$ . From the singularity, the total surface area of a cylindrical sector gradually increases from zero as  $\rho \to \infty$  or  $\rho \to 0$ . It means that the previouslyknown wormhole structure in considering the transverse  $S^2$  surface, does not appear in the full geometry. In fact, such a wormhole structure is a consequence of the dimensional reduction to the 4D Jordan frame. If we transform it to the 4D Einstein frame, the wormhole structure is naturally absent. We also point out that even in the 4D Jordan frame the proper length in the radial direction between the throat  $(\rho = \rho_+)$  and  $\rho = K$  is not infinite, but is finite.

While the  $\rho=K$  surface of the Schwarzschild black string is an event horizon, it is not true for the others. The outgoing-null rays emitted from the singularity take finite time to escape. In addition, there is no event horizon in the outer geometry either. Therefore, the singularity at  $\rho=K$  is naked. However, the lapse function becomes zero at the singularity, so the outgoing-null rays get infinitely redshifted.

Another interesting issue related with this hypercylindrical system would be investigating its stability. Although it is known that the Schwarzschild black string is unstable to perturbations and experiences the Gregory-Laflamme instability, the physical cause of the instability has not been revealed yet. While the Schwarzschild black string is described by only one parameter, the general hypercylindrical solution that we have now has two parameters. The complete knowledge of the role of the parameters, particularly the tension density, may help in understanding the physical cause of the black string instability. We will come back to this issue in future work.

#### Acknowledgments

This work was supported by the Basic Research Program of the Korea Science & Engineering Foundation under the grant No. R01-2006-000-10965-0 and Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARCSEC) (I.C.), in part by Grand Challenge Project at KISTI (G.K.), by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MOST) under the grant No. R01-2005-000-10404-0 (S.P.K.), and No. R01-2006-000-10651-0 (C.H.L.), and in part by the Asia Pacific Center for Theoretical Physics.

- R. Gregory and R. Laflamme, Phys. Rev. Lett. 70, 2837 (1993) [arXiv:hep-th/9301052]; Nucl. Phys. B 428, 399 (1994) [arXiv:hep-th/9404071].
- [2] G. T. Horowitz and K. Maeda, Phys. Rev. Lett. 87, 131301 (2001) [arXiv:hep-th/0105111].
- [3] S. S. Gubser, Class. Quant. Grav. 19, 4825 (2002) [arXiv:hep-th/0110193]; T. Wiseman, ibid. 20, 1137 (2003) [arXiv:hep-th/0209051]; M. W. Choptuik et al., Phys. Rev. D 68, 044001 (2003) [arXiv:gr-qc/0304085]; T. Harmark and N. A. Obers, JHEP 0205, 032 (2002) [arXiv:hep-th/0204047]; B. Kol, ibid. 0510, 049 (2005) [arXiv:hep-th/0206220]; T. Wiseman, Class. Quant. Grav. 20, 1177 (2003) [arXiv:hep-th/0211028]; T. Harmark and N. A. Obers, ibid. 21, 1709 (2004) [arXiv:hep-th/0309116].
- [4] T. Hirayama, G. w. Kang and Y. o. Lee, Phys. Rev. D 67, 024007 (2003) [arXiv:hep-th/0209181]; G. Kang and J. Lee, JHEP 0403, 039 (2004) [arXiv:hep-th/0401225].
- [5] T. Hirayama and G. Kang, Phys. Rev. D 64, 064010 (2001) [arXiv:hep-th/0104213].
- [6] D. Kramer, Acta Phys. Polon. B 2, 807 (1971).
- [7] A. Chodos and S. Detweiler, Gen. Rel. Grav. 14, 879 (1982).
- [8] D. J. Gross and M. J. Perry, Nucl. Phys. B 226, 29 (1983).
- [9] A. Davidson and D. A. Owen, Phys. Lett. B 155 (1985) 247.
- [10] M. Yoshimura, Phys. Rev. D 34, 1021 (1986); R. C. Myers, Phys. Rev. D 35, 455 (1987); T. Koikawa and K. Shiraishi, Prog. Theor. Phys. 80, 108 (1988); B. V. Ivanov, IC/88/390 (1988); IC/89/3 (1989); IC/89/4 (1989); S. W. Kim and B. H. Cho, Phys. Rev. D 40, 4028 (1989); T. Matos and N. Breton, Gen. Rel. Grav. 26, 827 (1994) [arXiv:gr-qc/9403022]; K. A. Bronnikov, Grav. Cosmol. 1, 67 (1995) [arXiv:gr-qc/9505020]; A. G. Agnese, A. P. Billyard, H. Liu and P. S. Wesson, Gen. Rel. Grav. 31, 527 (1999) [arXiv:gr-qc/9905037]; H. Liu and J. M. Overduin, Astrophys. J. 538, 386 (2000) [arXiv:gr-qc/0003034]; J. Ponce de Leon, Gen. Rel. Grav. 36, 923 (2004) [arXiv:gr-qc/0212058]; A. Coley and N. Pelavas, Gen. Rel. Grav. 38, 445 (2006) [arXiv:gr-qc/0510064]; K. Lake, Class. Quant. Grav. 23, 5871 (2006) [arXiv:gr-qc/0606005]; A. Feinstein, arXiv:gr-qc/0606054; J. P. de Leon, Class. Quant. Grav. 24, 1755 (2007) [arXiv:gr-qc/0701129].
- [11] C. H. Lee, Phys. Rev. D **74**, 104016 (2006) [arXiv:hep-th/0608167].
- [12] H. Elvang, T. Harmark and N. A. Obers, JHEP 0501, 003 (2005) [arXiv:hep-th/0407050].
- [13] J. Lee and H. C. Kim, arXiv:hep-th/0702085; arXiv:gr-qc/0703091.
- [14] H. Y. Liu, P. Wesson and J. Ponce de Leon, J. Math. Phys. 34, 4070 (1993).
- [15] A. Billyard and P. S. Wesson, Phys. Rev. D 53, 731

- (1996).
- [16] J. H. Traschen and D. Fox, .Class. Quant. Grav. 21, 289 (2004) [arXiv:gr-qc/0103106].
- [17] P. K. Townsend and M. Zamaklar, Class. Quant. Grav. 18, 5269 (2001) [arXiv:hep-th/0107228].
- [18] T. Harmark and N. A. Obers, JHEP 0405, 043 (2004) [arXiv:hep-th/0403103].
- [19] J. H. Traschen, Class. Quant. Grav. 21, 1343 (2004)
- [arXiv:hep-th/0308173].
- [20] J. Ponce de Leon, arXiv:gr-qc/0611082.
- [21] The same phenomenon arises in the cosmic p-brane solutions in Ref. [22].
- [22] R. Gregory, Nucl. Phys. B **467**, 159 (1996) [arXiv:hep-th/9510202].
- [23] In Ref. [11], the  $\rho=\rho_+$  surface was misled as an event horizon. We correct it here.